

Three-vortex configurations in trapped Bose-Einstein condensates

J. A. Seman^{1,*}, E. A. L. Henn¹, M. Haque², R. F. Shiozaki¹, E. R. F. Ramos¹, M. Caracanhas¹,
P. Castilho¹, C. Castelo Branco¹, G. Roati³, K. M. F. Magalhães¹, and V. S. Bagnato¹

¹*Instituto de Física de São Carlos – USP. C.P. 369, São Carlos – SP – Brazil – 13560-970*

²*Max Planck Institute for Physics of Complex Systems Nöthnitzer Strasse 38, 01187 Dresden, Germany*

³*LENS and Dipartimento di Fisica, Università di Firenze,
and INFN-CNR, Via Nello Carrara 1, 50019 Sesto Fiorentino, Italy*

We report on the creation of three-vortex clusters in a ^{87}Rb Bose-Einstein condensate by oscillatory excitation of the condensate. This procedure can create vortices of both circulation, so that we are able to create several types of vortex clusters using the same mechanism. The three-vortex configurations are dominated by two types, namely, an equilateral-triangle arrangement and a linear arrangement. We interpret these most stable configurations respectively as three vortices with the same circulation, and as a vortex-antivortex-vortex cluster. The linear configurations are very likely the first experimental signatures of predicted stationary vortex clusters.

PACS numbers: 03.75.Lm, 67.85.De

Introduction — Quantized vorticity being a key feature of superfluidity, vortex dynamics and configurations have long been central topics in the study of Bose-Einstein condensates (BECs). In the context of trapped atomic BECs, vortex dynamics has enjoyed a resurgence of interest because of new physical effects associated with the trap geometry. Vortices have been produced in BECs by transferring angular momentum either by manipulation of the quantum phase and the hyperfine state of the atoms [1] or directly by mechanical means [2, 3]. The majority of subsequent experiments have focused on externally rotated condensates, where vortices of the same circulation sign arrange into vortex lattices [4]. On the other hand, in BECs without external rotation, one can have vortices of both circulation signs coexisting. In a trap such combinations display fundamentally new effects, such as stationary structures [5, 6, 7]. For example, in pancake-shaped BECs, a stable linear configuration exists for vortex tripoles if the interaction is large enough [6, 7]. This linear arrangement contains an anti-vortex at the trap center and two vortices equally spaced on either side of the anti-vortex.

The purpose of this Letter is to report experimental evidence for the stability of specific three-vortex configurations in a trapped BEC. We create three-vortex states through an external oscillatory perturbation in a ^{87}Rb BEC held in a cigar-shaped magnetic trap. Our analysis of the three-vortex configurations clearly indicate the existence of stable linear structures. We interpret these as the vortex tripole structures of Refs. [6, 7] described above. We also find structures where the three vortices form an approximately equilateral triangle. We interpret these as three vortices with the same circulation.

In the absence of extensive experimental results on few-vortex dynamics and configurations in non-rotating trapped BECs, our results open up an intriguing direction of vortex studies. By realizing the linear configuration in a cylindrical geometry, we demonstrate that the

phenomenon of stationary vortex cluster configurations is robust well beyond the two-dimensional (2D) or pancake limit. In addition, the study of the dynamics and coexistence of vortices and anti-vortices is an essential ingredient for understanding evolution to turbulence as well as decay processes. More generally, interest in the interplay between vortex dynamics and confinement effects extends beyond cold-atom physics, since analogous issues with magnetic vortex/anti-vortex effects in ferromagnetic microstructures have attracted a great deal of attention in recent years [8].

Experimental results — The experimental setup and the vortex excitation procedure are described in detail elsewhere [3, 9]. Briefly, we use a ^{87}Rb BEC with 1×10^5 atoms held in a cigar shaped magnetic trap with frequencies $\omega_z = 2\pi \times 23$ and $\omega_r = 2\pi \times 207$ Hz. Vortices are generated by applying on the BEC an oscillating quadrupolar magnetic field whose amplitude is much smaller than the trap fields. As a function of field amplitude we observe an increasing number of vortices formed, up to the limit where a vortex tangle is observed [10].

For this study, we focus on images where a clear 3-vortex configuration is observed within the condensate core. Therefore, we have restricted the amplitude and frequency of the excitation for generating only this kind of structure. The excitation time was fixed at 20 ms, followed by an equilibrating time of 20 ms before releasing the atoms for free expansion. This is important because different excitation times can lead to different physical situations. Vortices are produced while the BEC is still held in the trap. For our analysis we will assume that the in-trap structure is maintained during the 15 ms of time-of-flight. In fact, this is a widely used assumption in experimental vortex studies.

We observe predominantly two types of configurations. The vortices are either distributed as a near-equilateral triangle, as in Fig. 1(a), or as a near-linear array, as in Fig. 1(b). We analyze the geometrical distribution by

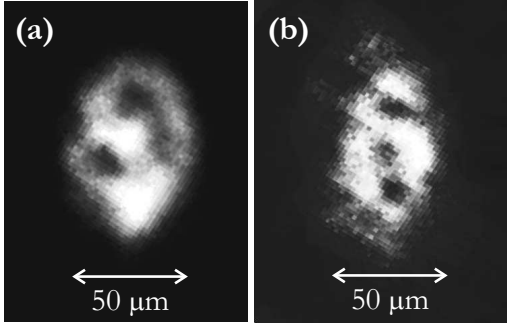


FIG. 1: Atomic optical density images showing configurations of stable vortices forming (a) an equilateral triangle, or (b) a linear array. Images were taken after 15 ms of free expansion.

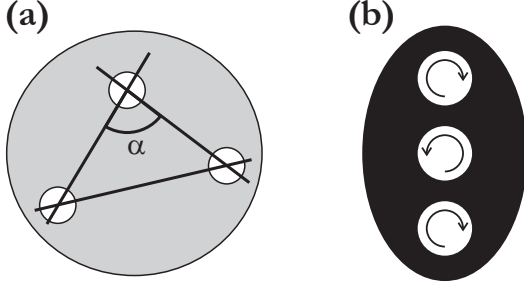


FIG. 2: (a) Definition of the largest internal angle α . (b) Schematics of the tripole configuration of vortices, arrows indicate the vortex circulation direction.

recording the largest internal angle, α , of the triangle whose vertices are the vortex positions, as sketched in Fig. 2(a). The histogram of Fig. 3 summarizes our results of more than 60 independent measurements, showing the relative frequencies of observed values of α . Two types of stable configurations have a clear prominence over the others, namely, an equilateral triangle type where the angle α is around 60° , and a linear array where $\alpha \sim 180^\circ$.

Interpretation — In the inset to Figure 3, we show the expected distribution of the largest angle α when the three vortices are distributed at random positions within the visible BEC core, taken to be an ellipse with aspect ratio ~ 1.5 . The intermediate $\alpha \in [100^\circ, 140^\circ]$ arrangement would be expected to be almost as numerous as the equilateral-like $\alpha \in [60^\circ, 100^\circ]$ arrangement. To explain the fact that we get much fewer intermediate configurations, we consider the *dynamics* of three vortices with the same circulation. For simplicity, our discussion and the simulations of Figure 4 use a circular trap, but this is sufficient for the present discussion.

Three vortices placed symmetrically around the trap center ($\alpha = 60^\circ$) in a nonrotating condensate simply precess around the center, maintaining the equilateral configuration. If the initial configuration is moderately distorted from this symmetric configuration, the dynamics of the triangle shape is also moderate, so that α does

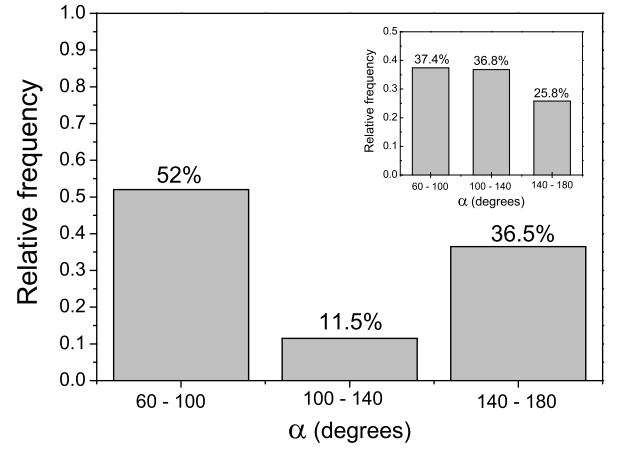


FIG. 3: Observed relative frequency of 3-vortex configurations as a function of the angle α . The inset shows the expected distribution of α when the vortices are distributed at random positions.

not vary too much. Thus, a significant fraction of the configurations starting with $\alpha \in [60^\circ, 100^\circ]$ stay within this range during the precession dynamics (Fig. 4A,B).

However, when the initially created vortex positions are such that $\alpha \in [100^\circ, 140^\circ]$, the triangle formed by their positions changes shape as the vortices precess, so that the oscillations of α takes it in and out of the $\alpha \in [60^\circ, 100^\circ]$ range (*e.g.*, Fig. 4C). Thus the preponderance of near-equilateral configurations in our experiment reflect the fact that there are many $\alpha \in [60^\circ, 100^\circ]$ configurations which are ‘stable’ in the sense of maintaining the triangle shape while precessing.

While the equilateral arrangement is reminiscent of the Abrikosov lattice observed by many authors [11], and of the three-vortex case studied in Refs. [12] and [13], our measurements explore the rather different physics of non-rotating condensates. Our results reveal that in the situation without external rotation the equilateral arrangement also plays a special role, through maintaining its shape during precession.

From similar arguments (Fig. 4D), if all vortices have the same circulation direction, we would expect a smaller frequency of near-linear configurations with $\alpha \in [140^\circ, 180^\circ]$, compared to the random-distribution case of Figure 3 inset. In our experimental data, however, the linear array is more frequent than the random-distribution prediction. This indicates that in the linear configurations the vortex circulations are not all of the same sign.

Refs. [6, 7] have shown that a tripole configuration like the one shown in Fig. 2(b) can be a stable configuration. For a pancake-shaped condensate, the vortex tripole is metastable when the interaction strength is above a certain value [7]. In this situation, the interaction strength is expressed by the effective 2D coupling parameter \tilde{g} ,

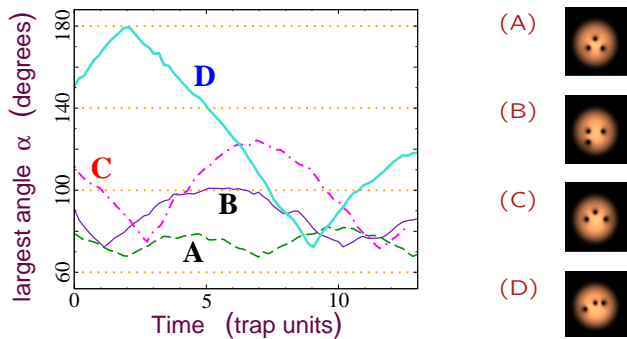


FIG. 4: (Color online.) Evolution of the largest angle α , in Gross-Pitaevskii simulations starting from various three-vortex configurations in a circularly trapped two-dimensional BEC. Initial configurations are shown on right.

defined as

$$\tilde{g} = \sqrt{8\pi} \frac{Na}{a_{\perp}}, \quad (1)$$

where a is the scattering length, N is the number of particles, and a_{\perp} is the oscillator length in the direction perpendicular to the plane considered to be quasi-2D. Following Ref. [7] the condition for the tripole configuration to be stable is $\tilde{g} \geq 108$. In our experimental geometry, the perpendicular direction is one of the radial directions, $a_{\perp} = a_r = \sqrt{\hbar/m\omega_r}$, so that $\tilde{g} \approx 200$. Thus, from quasi-2D arguments, we expect the linear tripole to be metastable in our setup, which supports our interpretation that the predominance of linear configurations is due to the tripole of Refs. [6, 7]. The fact that our setup is not really quasi-2D indicates that the results of Ref. [7] are valid well beyond the pancake-shape limit.

It is expected [7] that the tripole survives for a time of the order of ω_0^{-1} , which for our case is about 10 ms, well within the 20 ms of waiting time before releasing the atoms from the magnetic potential. Thus tripole decay has a chance to reduce the number of observations of tripole configurations, as discussed further below. In fact, for a very long waiting time after excitation, observation of collinear 3-vortex configuration seems to be scarcer but fluctuation does not allow us to be more quantitative in this regard.

The experimental method used here to produce vortex states has basically the same probability to produce both circulation sign vortices. Such equal chances come from the fact that the oscillatory field introduced by the extra coils modulates the equilibrium position as well as the overall trap shape introducing features that result in a vortex state of both circulation signs, as discussed in Ref. [3]. The equal chances for vortex and anti-vortex formation implies that in many of the observed distributions a collection of two vortices and one anti-vortex can occur. Since such a combination has a metastable

linear structure, this would explain the prominence of near-linear structures in Figure 3.

Relative abundance of configurations — If equal chances to produce vortices and anti-vortices are considered, we would expect that the probability P_{line} to excite a mixture containing one vortex with certain sign of circulation and two vortices with the opposite one would be higher than the probability $P_{triangle}$ to produce three vortices with the same sign. Considering the data presented in Fig. 3, we have $P_{line} \approx 0.37$ and $P_{triangle} \approx 0.52$. From a simple probability consideration and assuming the production of a sequence of vortices as independent events, we would have the probability of having three vortices of the same sign as 0.25 while for having one of opposite sign among them as 0.75. That would produce a ratio 1 : 3 and not 1 : 0.7 as observed in our experiment. This mismatch is probably due to the complex dynamics occurring after vortex nucleation. If the surviving chances were equivalent, we would expect more events showing tripole configuration than triangular configuration. Decay mechanisms may, however, modify completely such a expectation. Vortex tripole may have a vortex/anti-vortex pair annihilated. Therefore, this may be indicative of higher instability for the situation composed of two vortices and one anti-vortex. For times longer than $\sim \omega_0^{-1}$, the tripole tends to decay, for example through one vortex migrating to the borders of the condensate generating surface modes, and leaving behind a vortex dipole [7].

Let us consider that the tripole configuration decays exponentially, with time constant $\omega_0^{-1} \approx 10$ ms, to a dipole configuration. In that case, supposing that the tripole was generated, the probability of observing it after the 20 ms of waiting time decreases by a factor of $e^{-2} \approx 0.135$. Let us additionally consider that the triangular configuration (with all vortices of the same sign of circulation) is preserved during the waiting time. Taking all this into account, if a configuration with three vortices is initially produced, the probability of observing the triangle remains as 0.25, the probability of observing a tripole is 0.10 and the probability of observing a dipole is 0.65 (*i.e.*, the expected configuration after the tripole decay). If from these observations we only consider the configurations with three vortices, we shall have that $P_{triangle} \approx 0.715$ and $P_{line} \approx 0.285$. This is much closer to the observed values in our experiment. Since this is not the unique decay mechanism (vortex/anti-vortex pairing may also be important) the tripole configuration should be undercounted. That may explain the relative values in the histogram of Fig. 3.

Discussion — In summary, we have observed 3-vortex cluster configurations where two different types are the most frequent: the equilateral triangle geometry and the linear array. We have argued that the triangular cluster type is formed predominantly by vortices with the same sign of circulation while the linear array is numer-

ous because of configurations where the middle vortex has opposite sign.

The present experiments are loosely related to the work of Ref. [14] where different vortex configurations were created by merging multiple BECs. Almost linear or triangle structures can also be seen in their data. Our results complement those of Ref. [14], using a quite different vortex formation mechanism.

Vortex-antivortex dynamics in confined geometries is of broader interest than superfluid gases, since similar dynamical issues are currently under intense study in the magnetic microstructure context [8]. Within the field of cold atomic gases, our experiment features a distinct manner for creating vortices of both circulation in the same condensate. Our results highlight several poorly understood aspects of vortex physics, in particular involving non-equilibrium dynamics. Since the imaging is performed after the released condensates have expanded and have undergone aspect ratio inversion, there are several issues related to the expansion dynamics. It is not known in any detail whether or not the presence of vortex clusters affects the inversion dynamics, or how the vortex configurations are rearranged as the cloud undergoes anisotropic expansion.

Another issue raised by the current experiments is the stability of the tripole configurations in a cylindrical rather than pancake-shaped condensate, since the theoretical studies [5, 6, 7] concern flat condensates. The abundance of the linear tripole configuration seen here suggests that the scissor oscillations used to excite the condensate may stabilize the tripole configuration, perhaps counteracting the dynamical instability discussed in Ref. [7]. These effects are clearly of basic interest but are yet to be theoretically analyzed in detail. In addition, investigation of the dynamics of superfluids containing both vortices and anti-vortices may be a fruitful alternative way to understand basic decay mechanisms of quantum turbulence and other poorly understood phenomena in atomic BECs.

Special thanks to C. Salomon and W. D. Phillips for fruitful discussion. This work was supported by FAPESP (research program CEPID/CEPOF), CAPES and CNPq (research program INCT).

* Electronic address: jorge@ursa.ifsc.usp.br

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